

Some fascinating developments in Mathematics and Music

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Mathematics and Music have been intertwined at least since Pythagoras. However, it is a common misconception to believe that the mathematics of music are elementary. Leibniz's famous saying « Musica est exercitia arithmeticae » is nowadays reductive and misleading, since the understanding of music involves most domains of mathematics, not only numbers but also Category theory, differential calculus, word theory, general and linear algebra, to name a few. This talk presents several fairly recent examples: an empirical discovery of minimalist composer Tom Johnson about doing rhythmic canons with the simple motif $xx..x$ cannot be proved without Galois theory of finite fields; Beethoven used a Hamiltonian cycle in his XIXth symphony years before they were invented, and centuries before it was noticed, leading to an enumeration of the 262 cycles in the Tonnetz — the graph of minor and major triads in 2009 — now available to composers; and the generation of musical intervals by octave and fifth, such as Plato alluded to in the *Timaeus*, produces an original theorem on palindroms in an infinite word

abcbcdccdcceccceceeece...

extrapolating the pythagorean scale.

Not only is the « unreasonable efficiency of mathematics » at work in music as it is in other sciences: the organic relationship between these two extremely deep fields of knowledge and experience enable musical ideas to permeate mathematics and help unravel new ideas and results.

