COMPLEXITY IN THE ARTS AND SCIENCES:
" A CASE OF "ART IMITATES LIFE"

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" COLLOQUIUM ARTS MEETS SCIENCE"

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EUROPEAN ACADEMY
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Is there Complexity underneath the earth?

"Cave of the Lakes", Peloponnese, Greece

Is this Complexity beautiful?

Is there Complexity in the sky?

Have artists been inspired by its beauty?

SUMMARY OF THE PRESENTATION:

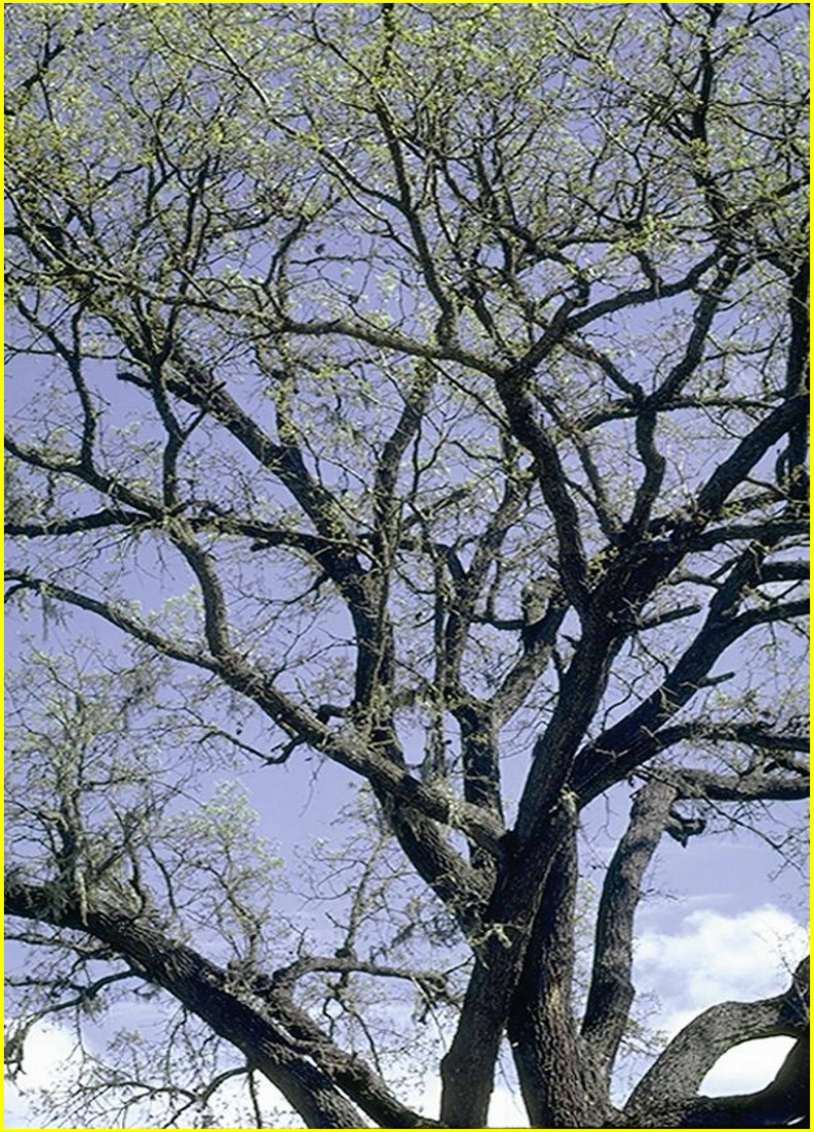
1. A Mathematics Approach:

- Brief Introduction to Fractal Geometry
- Fractal analysis of paintings by Piet Mondrian (1872 – 1944).

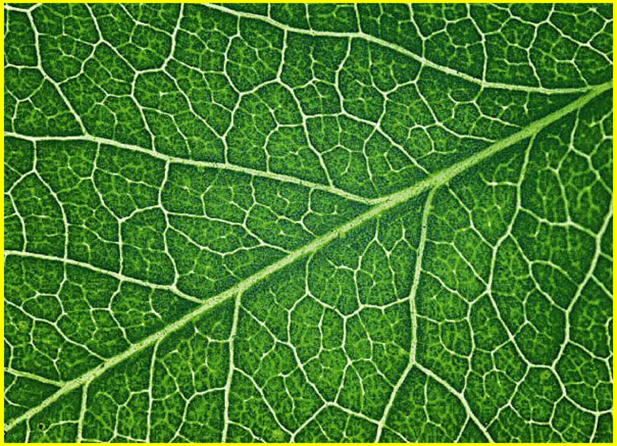
2. A Physics Approach:

- Applications of Complexity and Permutation Entropy to the history of art paintings and to different artistic styles over 1000 years!

3. Questions and Discussion



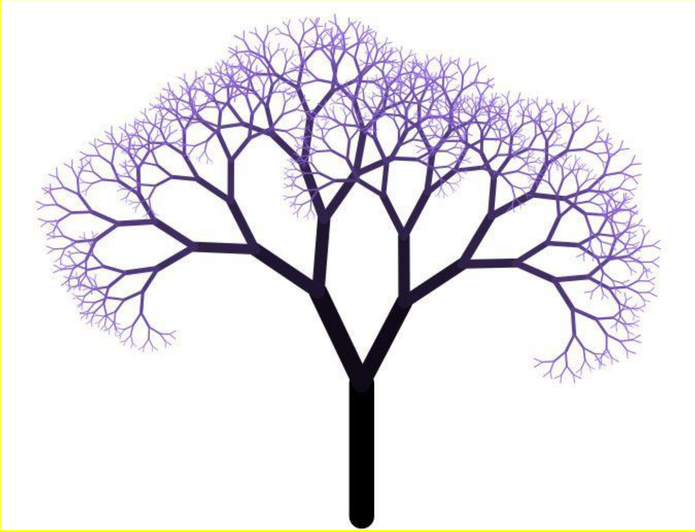
What is it that impresses us about the branching of a tree?



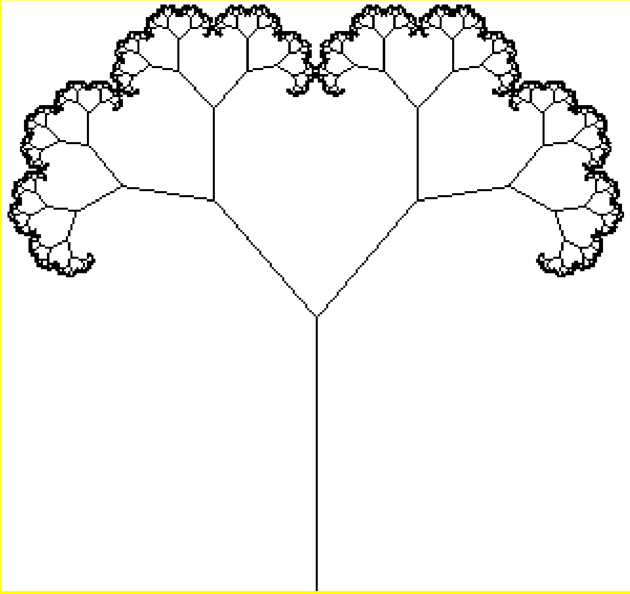
or the “structure” of its leaves?

How can we understand their complexity?

The Physics Nobel Laureate Richard Feynman once said: "What I cannot create, I do not understand."



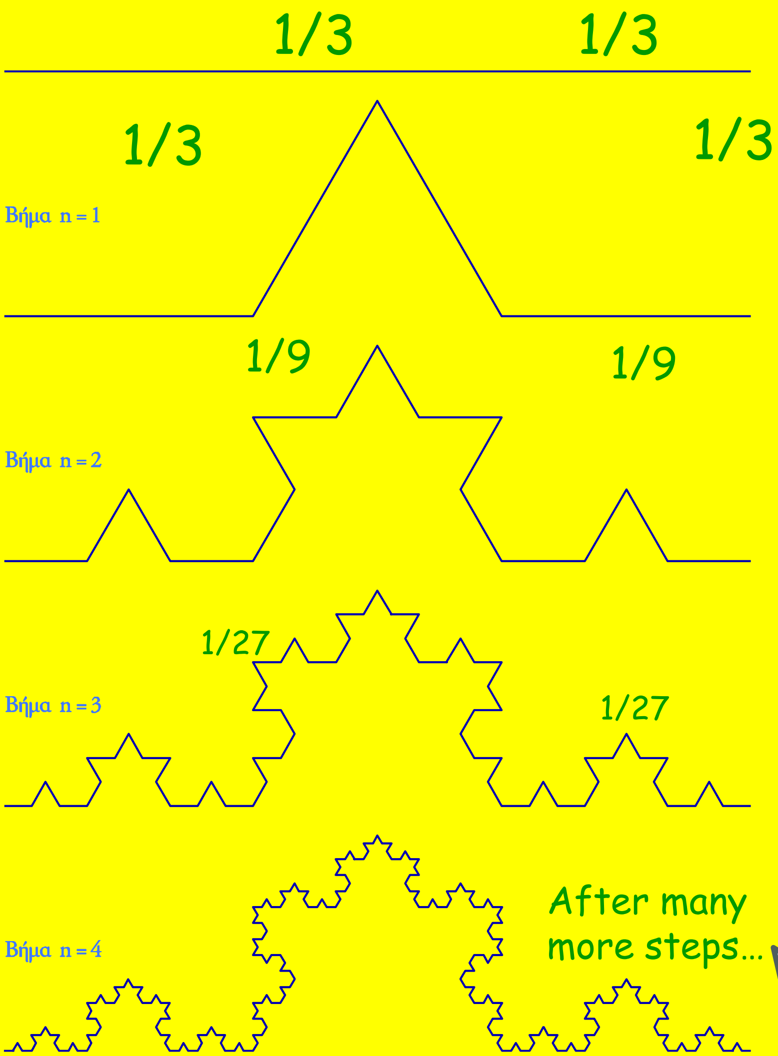
Observe in this design a self-similarity in the way smaller branches bifurcate out of a bigger branch so that they are smaller by a scaling factor (say ½).



Now, let's draw this "tree": Shorten the branches by 1/2 at every bifurcation, and rotate them (on both sides) by 45^{o} and you find..

Not so bad! But, can we develop more sophisticated mathematical approaches to describe this type of complexity?

Yes, using the theory of **Fractals!**



We can generate new objects by adding pieces of smaller and smaller scale, say $(1/3)^n, \quad n = 1, 2, 3, \dots$

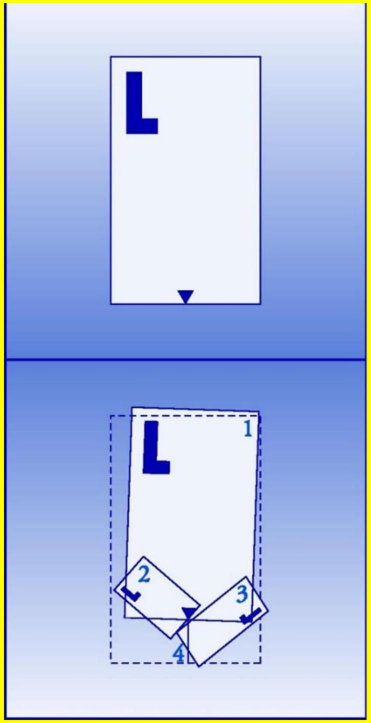
and thus create a mathematical "coastline", of greater and greater complexity!

This is the famous Koch curve, which "lives" in a world of non-integer dimension D = 1.26, greater than 1 but smaller than 2!

What have we done?
We managed to include an infinite amount of information in a very short space!

Now let's become more sophisticated and construct a fractal fern as follows:

After many iterations…



What have we done? We have constructed the famous Barnsley's fern using repeatedly the above transformation of 4 parallelograms one large (1) and three smaller ones (2,3 and 4).

Is there anything "beautiful" about Fractals?

Apparently, some artists thought so…

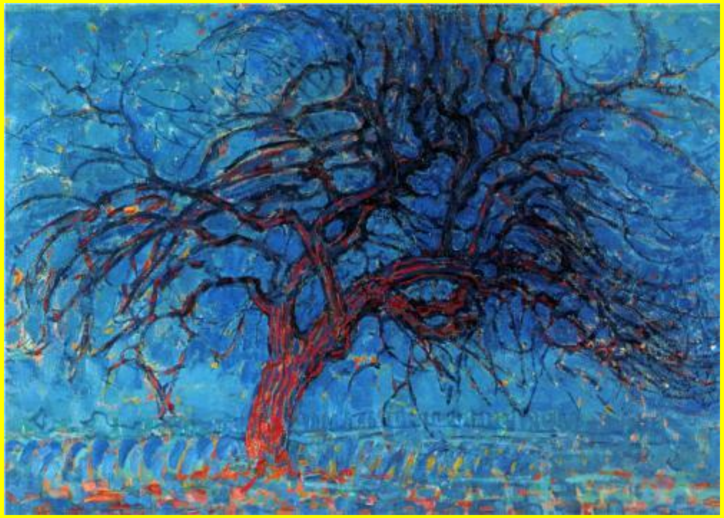
In a paper in Nature 399 (1999) R.P. Taylor, A. P. Micolich, claimed that many paintings by the great American painter Jackson Pollock have fractal properties, with fractal dimension D that evolved from near 1.0 in 1943 to 1.7, by the year 1953 !

"Alchemy" (1947)

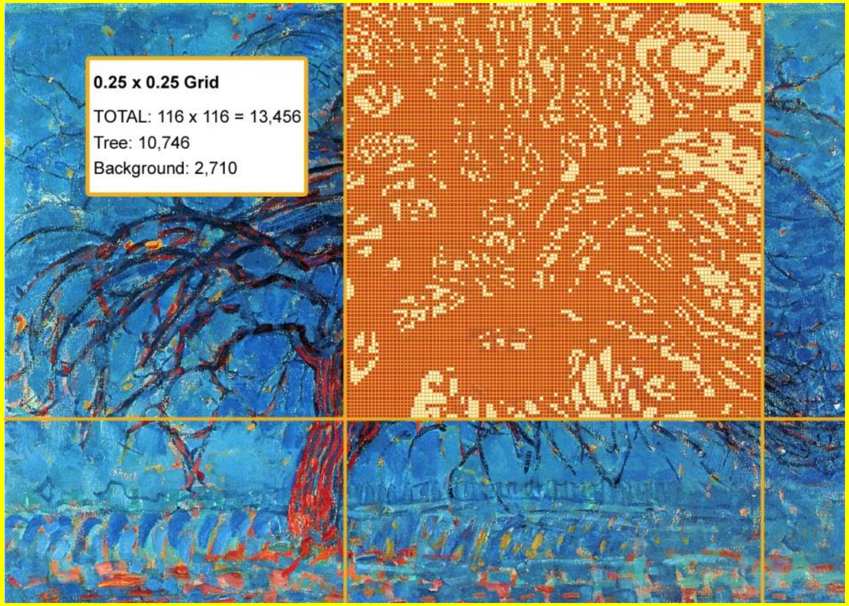
"Out of the Web" (1949)

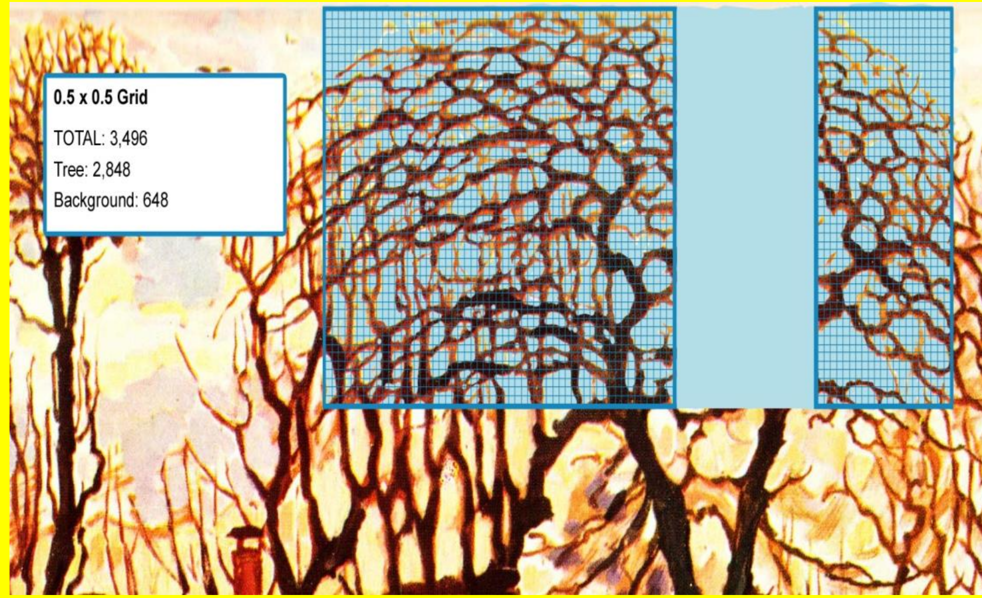
Now, fractality can be inferred only if one is able to examine it in detail, using a sequence of scales that become smaller down to as small a size as possible…

We thus decided in the paper T. Bountis, A.S. Fokas and E. Psarakis, Intern. J. of Arts and Technology, vol. 10(1), (2017) to perform a careful mathematical analysis of two Tree Paintings by Piet Mondrian (1872 – 1944) using methods of fractal geometry.



The Red Tree (left, 1910) and The Farm near Duivendrecht (right, 1916)





We "covered" parts of our paintings by "grids" of small squares of sides l_1 $=0.75 > l_2 =0.5 > l_3 =0.25$ …, counted their numbers $N(l_1) < N(l_2) < N_3(l_3)$ … and looked for the power D that produces a near equality for their total "measure" M in the relation:

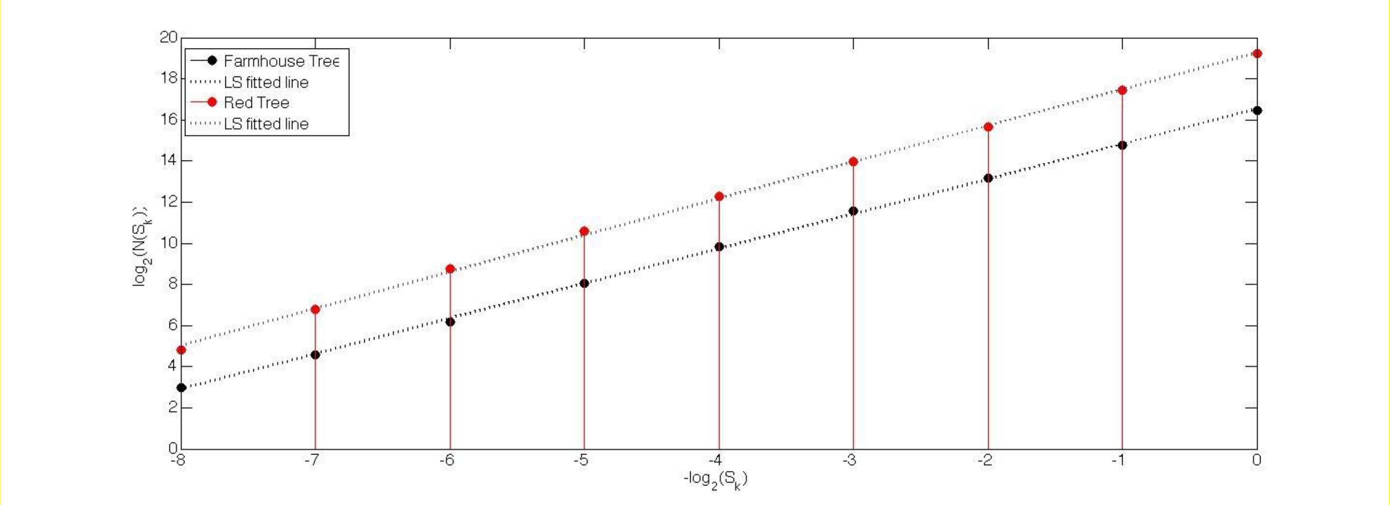
$$M \approx N(l_1) l_1^D \approx N(l_2) l_2^D \approx N(l_3) l_3^D \approx \dots$$

We thus arrived at a first estimate of the fractal dimension, which turned out to be for both paintings $D \approx 1.7 - 1.8$.

We then evaluated the sequence of dimensions

$$D_k = \frac{\log(N(S_k)) - \log M}{\log(S_k^{-1})} \quad , \quad k = K-1, \ldots, 2, 1$$

as an optimization problem using a technique called linear regression. Plotting a graph of the above quantities, we found nearly equal slopes for the two paintings:



which leads us to the conclusion that both paintings have nearly equal dimension $D \approx 1.75$.

The Physical Quantities of Entropy and Complexity

The "Entropy" H quantifies the degree of disorder in the pixel arrangement of an image: Values close to 1 indicate that pixels appear at random, while values close to zero indicate that pixels appear almost always in the same order.

Regular images are expected to have small entropy values, while images exhibiting less regularity (such as Pollock's drip paintings) are characterized by large values of entropy.

The statistical complexity C, on the other hand, measures the "structural" complexity" of an image: It is zero for the extremes of order and disorder in the pixel arrangement, and it is positive when an image contains more complex spatial patterns.

The joint use of the values of C and H offer useful information on a "complexity–entropy plane".

In the paper "History of art paintings through the lens of entropy and complexity", Higor Y. D. Sigakia, M. Perc and H. V. Ribeiroa, PNAS 115 (37) (2018) the authors studied paintings of 2000 artists, for 100 different styles, over 1000 years!

- Each painting was converted into a matrix whose elements are the average values of the shades of red, green, and blue.

- The authors calculated two physical measures using the probability distribution of 24 color patterns among the images: the normalized permutation entropy H (see paper [1] (below) and the statistical complexity C = H·D (see paper [2] below), where H quantifies disorder and D is a measure of "disequilibrium".

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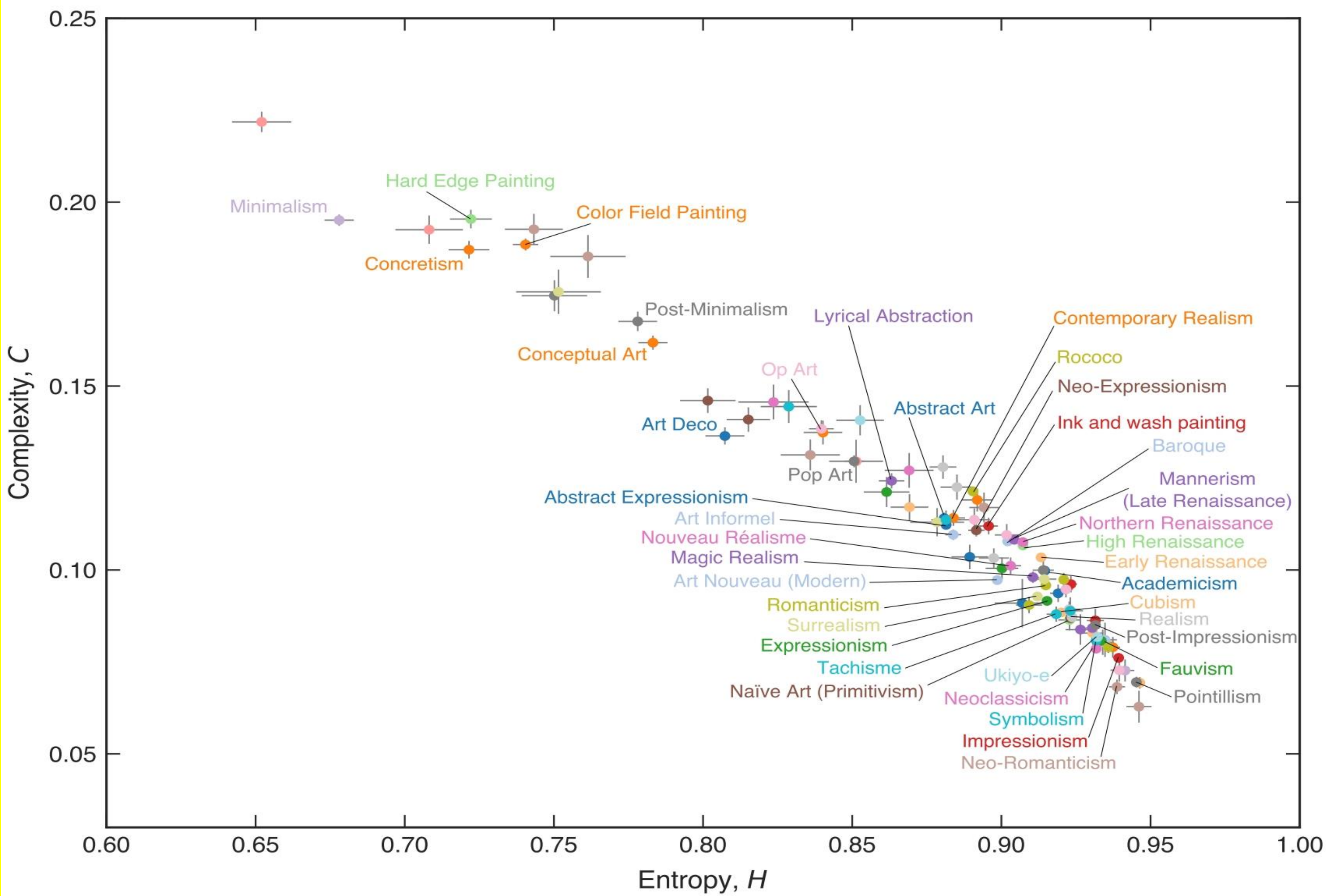
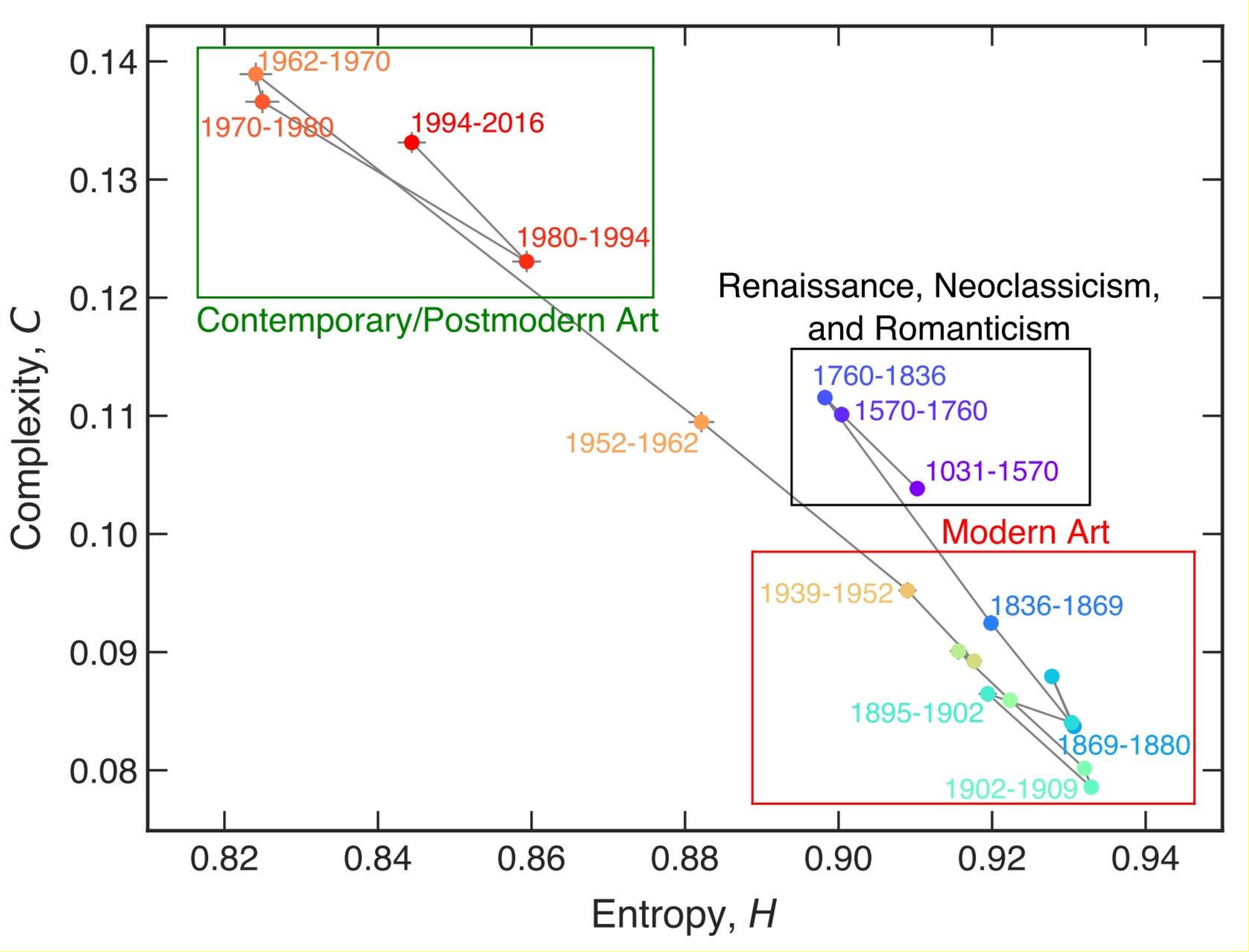
[1] Bandt C, Pompe B (2002) Permutation entropy: A natural complexity measure for time series. Phys Rev Lett 88:174102.

[2] Lopez-Ruiz R, Mancini HL, Calbet X (1995) A statistical measure of complexity. Phys Lett A 209:321–326.



"Proximity" of different artistic styles in a complexity - entropy plane,of 41 artistic styles with nearly 500 paintings for each style.



Grouping of different "art styles" through the history of painting, plotted on the H – C plane. Observe the difference in the years in each group!

Questions:

- Can one argue that "complexity" adds a kind of "inherent beauty" to a work of art?
- Based on complexity vs. entropy studies is it justified to classify different artistic styles over the centuries?
- Can we use concepts of mathematics and/or physics to study other forms of art, like sculpture or music for example?